Urban Vehicular Multipath Detection Using Multiple Antennas and Reliability Analysis

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BIOGRAPHY

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ABSTRACT

One of the most important issues when using GPS for urban vehicular navigation is the reliability of the position solution, which usually depends on the nature of the environment. In particular, the presence of urban canyons and foliage can cause significant degradation in satellite visibility as well as high multipath. The objective of this paper is to assess the impact of using multiple antennas along with some statistical reliability measure to detect blunders on pseudorange measurements, such that blunders can be rejected before they contaminate the estimated vehicle positions. One of the properties of multipath is that it decorrelates rapidly as a function of distance, so antennas spaced at least 0.5 m apart may be subjected to different multipath conditions making detection possible. The impact of using constraints between various antennas is also addressed. Land tests were conducted with four antenna/receivers in Calgary under various environments including open sky, urban canyon and dense foliage conditions. Results of the multipath blunder detection technique, when applied to the field data, are presented and discussed. An improvement of 10%-40% in position accuracy was achieved under different conditions.

INTRODUCTION

The Global Positioning System (GPS) has made land navigation applications affordable and dependable. However there are many situations where a GPS solution is either unavailable or unreliable. The first case occurs when GPS signals do not reach the antenna due to shading effects resulting from high rise buildings and underpasses present in an urban environment. The second situation arises from poor satellite geometry and the multiple reflection of signals. Although errors due to Selective Availability (SA), ionosphere, troposphere, multipath and receiver noise limit the achievable accuracy (Parkinson, 1994), the use of the differential GPS (DGPS) technique improves both accuracy and integrity although it does not reduce multipath (Parkinson and Enge, 1995).

Several multipath mitigation techniques have been developed such as the Narrow Correlator^{TM} (van Dierendonck et al., 1992) which has 0.1 chip spacing and a larger bandwidth at the IF and provides good long delay multipath mitigation. Similar technologies like MEDLL^{TM} (Van Nee, 1995), Edge Correlator™ (Garin et al, 1996), Strobe Correlator^{TM} (Garin and Rousseau, 1997) use the correlator based approach to mitigate mulitpath. However, code multipath errors can be as large as several tens of meters even with currently available state-of-the-art receiver technologies, and cannot be removed through differential positioning due to its highly localized nature (Braasch, 1994).

Code multipath is typically the most significant error source for differential vehicular navigation applications, especially in urban and semi-urban areas with buildings and trees. The behavior of code multipath in dynamic scenarios is very different from the static case. Cannon and Lachapelle (1992) did a detailed analysis of multipath in high performance receivers for kinematic applications. In this case, the position of various multipath sources change rapidly and therefore the total multipath signal is difficult to model. One of the properties of multipath is that it decorrelates rapidly as a function of distance between the reflecting source and the receiving antenna. Therefore, two antennas spaced at least 0.5 m apart, may be subjected to different multipath conditions even in a dynamic environment. Hence, a configuration of four antennas placed one meter apart from each other is used for analysis. Ray (2000) gives a detailed description of code and carrier multipath and their respective characteristics.

Parkinson and Axelrad (1988) demonstrated the concept of using reliability theory to detect gross blunders in GPS pseudorange measurements. Also, from classical least-squares adjustment theory (Leick, 1995), better performance can be achieved applying constraints. Hence, an attempt has been made to make use of both techniques for reliable navigation in an urban environment. This paper addresses the issues of combining multiple antennas and concepts of reliability together to detect the presence of multipath.

METHODOLOGY

An approach to detect multipath is to treat the errors as blunders and then define a statistical test to detect corrupted measurements. This method is similar to the RAIM algorithm proposed by Parkinson and Axelrad (1988). The defined statistical test assumes only one blunder to be present at any given instant, however this assumption may not be always true. Therefore, a reliability measure based on internal and external reliabilities is computed which can be used as a quality indicator. If the statistical test identifies the blunder then the particular measurement is eliminated from the estimation process.

In addition to the statistical test, measurements from several antennas may be combined using constraints before solving for the parameters. This is also expected to improve the reliability. Some of these concepts are discussed below.

RELIABILITY ANALYSIS

Reliability refers to the ability to detect measurement blunders and to estimate the effects of undetected blunders on a solution (Ryan et al., 1999). There are two kinds of reliability, namely internal and external (Krakiwsky and Abousalem, 1995). Internal reliability is defined as the minimum detectable measurement blunder determined from a statistical test, while external reliability refers to the impact that an undetected blunder can have on an estimated parameter.

In order to detect a blunder on an observation, a statistical test is performed with the underlying assumption that the residuals are normally distributed. Such a statement about the probability distribution of the population is called a statistical hypothesis. For every null hypothesis (H_0), an alternate hypothesis (H_1) exists. A hypothesis is tested by drawing a sample from the population, computing the value of a specific sample statistic, and then making the decision as to accept or reject the hypothesis based on the value of the statistic (Mikhail and Gracie, 1998). The hypothesis H_0 cannot result in a certain definite outcome, as the test is based on a data drawn from a sample population and not from the entire population. Hence, four possible outcomes can occur, namely

1. H_0 is accepted, when H_0 is true
2. H_0 is rejected, when H_0 is true
3. H_0 is accepted, when H_0 is false
4. H_0 is rejected, when H_0 is false

If outcome (1) or (4) occurs, then no error is made and the correct action has been taken. Outcome (2) is known as a *Type I error* and outcome (3) is referred to as a *Type II error*.

A Type I error occurs when a valid observation is rejected and the probability associated with the occurrence of a Type I error is denoted as α. A Type II error occurs when a bad observation is accepted and the probability associated with this happening is β. Figure 1 graphically shows the relationship between Type I and II errors. The non-centrality parameter $\left(\sqrt{w_0}\right)$, which is also the bias in the standardized residuals, can be determined by selecting values for α and β from Table 1. Baarda (1968) introduced this concept of fixing the size of the model error that can be detected at a certain probability level by a certain test.

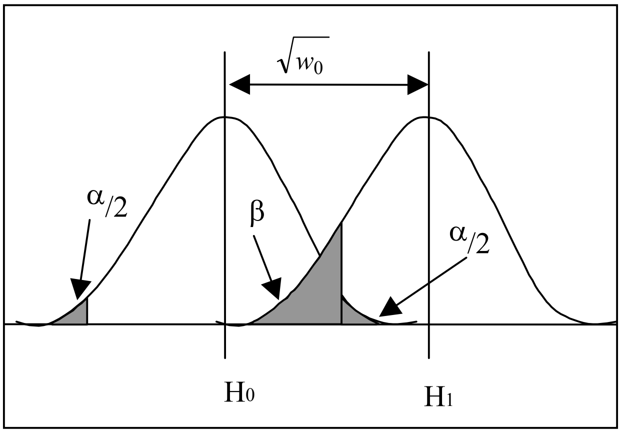

Figure 1 : Type I and Type II Errors with Non Centrality Parameter

Table 1: Non Centrality Parameter (Leick, 1995)

α	β	$\sqrt{w_0}$
5.0%	20%	2.80
2.5%	20%	3.10
5.0%	10%	3.24
2.5%	10%	3.52
0.1%	20%	4.12
0.1%	10%	4.57

Once $\sqrt{w_0}$ is computed, the following statistical test is performed:

$$\tilde{r}_i = \frac{\hat{r}_i}{\sigma_{\hat{r}_i}} \qquad (1.1)$$

Where,

$\tilde{r}_i$ is the normalized residual
$\hat{r}_i$ is the residual of the measurement, and
$\sigma_{\hat{r}_i}$ is the standard deviation of the residual

The smallest blunder that can be detected by the statistical test is given by equation 1.2 and is also referred to as the Marginally (or Minimum) Detectable Blunder (MDB), see Salzmann (1991) for example.

$$MDB_i = \sigma_{li}\sqrt{\frac{w_0}{g_i}} \qquad (1.2)$$

Where

i is the observation number
$\sqrt{w_0}$ is the non-centrality parameter
g_i is the redundancy number of the ith observable, and
σ_{li} is the standard deviation of the ith observable

The probability density of the residuals is $n(\xi;0,1)$ which is a standard normal density with mean of zero and a variance of one. The redundancy matrix g_i is given as

$$g_i = (C_{\hat{r}} C_l^{-1})_{ii} \qquad (1.3)$$

Where,

$C_{\hat{r}}$ is the covariance matrix of the residuals, and
C_l is the covariance matrix of the observations

Once the MDB for each observation has been calculated, the impact of this blunder on the parameter space, which provides a measure of the expected error on the parameters, is given by equation 1.4. This is also referred as External Reliability.

$$\Delta\hat{\delta}_i = -C_{\hat{x}} A^T C_l^{-1} \nabla_0^i \qquad (1.4)$$

Where,

$C_{\hat{x}}$ is the covariance matrix of the parameters
A is the design matrix
∇_0^i is a column vector containing all zero's except for the MDB in the ith position.

In the current application, measurements are differentially corrected, and as a result, only multipath errors and receiver noise are present. The receiver noise has a normal distribution whereas multipath errors in kinematic situation are random in nature (Nayak, 2000a). Hence, the residuals can be assumed to be normally distributed. However, if there are multipath errors, then the residuals will be biased and can be detected by the statistical test. This test actually eliminates blunders and does not distinguish multipath errors from other errors like integrity failures.

Only one blunder per antenna is considered to exist at any given time. Though this assumption appears insufficient, the fact is that maximum multipath is observed in urban conditions where satellite visibility is poor. By eliminating the measurement with the maximum multipath error, the reliability is expected to improve substantially.

To achieve better reliability, additional observations in the form of constraints between the antennas can be applied.

CONSTRAINTS

If two or more antennas are present, then fixed distance constraints between the antennas can be used. To do this, the distance between the antennas has to be measured *a priori*. The model used for a fixed baseline constraint is given by

$$f_{BL} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \qquad (1.5)$$

Where $(x,y,z)_1$ and $(x,y,z)_2$ are the WGS84 coordinates of the two antennas.

The design matrix for this constraint, which is of dimension 1xu (where u is the number of parameters), is

$$A = \left[\frac{\partial f}{\partial \phi_1} \frac{\partial f}{\partial \lambda_1} \frac{\partial f}{\partial h_1} 00000 \frac{\partial f}{\partial \phi_2} \frac{\partial f}{\partial \lambda_2} \frac{\partial f}{\partial h_2} 00000.....0 \right] \qquad (1.6)$$

and

$$\frac{\partial f}{\partial \phi} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \phi} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \phi} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial \phi} \qquad (1.7)$$

$$\frac{\partial f}{\partial \lambda} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \lambda} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \lambda} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial \lambda} \qquad (1.8)$$

$$\frac{\partial f}{\partial h} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial h} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial h} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial h}. \qquad (1.9)$$

If the approximate coordinates of the two antennas are known then the following can be derived (Cannon, 1991):

$$\frac{\partial f}{\partial \phi_1} = \frac{-(R_N+h_1)\sin\phi_1\cos\lambda_1(x_1-x_2)}{d_{12}} + \frac{-(R_N+h_1)\sin\phi_1\sin\lambda_1(y_1-y_2)}{d_{12}} + \frac{[R_N(1-e^2)+h_1]\sin\phi_1(z_1-z_2)}{d_{12}}$$

$$\frac{\partial f}{\partial \phi_2} = \frac{(R_N+h_2)\sin\phi_2\cos\lambda_2(x_1-x_2)}{d_{12}} + \frac{(R_N+h_2)\sin\phi_2\sin\lambda_2(y_1-y_2)}{d_{12}} + \frac{[R_N(1-e^2)+h_2]\cos\phi_2(z_1-z_2)}{d_{12}}$$

$$\frac{\partial f}{\partial \lambda_1} = \frac{-(R_N+h_1)\cos\phi_1\sin\lambda_1(x_1-x_2)}{d_{12}} + \frac{(R_N+h_1)\cos\phi_1\cos\lambda_1(x_1-x_2)}{d_{12}}$$

$$\frac{\partial f}{\partial \lambda_2} = \frac{(R_N+h_2)\cos\phi_2\sin\lambda_2(x_1-x_2)}{d_{12}} + \frac{(R_N+h_2)\cos\phi_2\cos\lambda_2(x_1-x_2)}{d_{12}}$$

$$\frac{\partial f}{\partial h_1} = \frac{\cos\phi_1\cos\lambda_1(x_1-x_2)}{d_{12}} + \frac{\cos\phi_1\sin\lambda_1(y_1-y_2)}{d_{12}} + \frac{\sin\phi_1(z_1-z_2)}{d_{12}}$$

$$\frac{\partial f}{\partial h_2} = \frac{-\cos\phi_2\cos\lambda_2(x_1-x_2)}{d_{12}} + \frac{-\cos\phi_2\sin\lambda_2(y_1-y_2)}{d_{12}} + \frac{-\sin\phi_2(z_1-z_2)}{d_{12}}$$

Where,

$$R_N = \frac{a}{\sqrt{1-e^2\sin^2\phi}}$$

$$d_{12} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

a is the WGS84 semi-major axis, and

e^2 is the ellipsoidal eccentricity

IMPLEMENTATION

The C^{3}NAV™ (Combined Code and Carrier for GPS NAVigation, Cannon and Lachapelle, 1995) software developed at the U of C. was modified to become MATNAV (Multiple AnTenna NAVigation), which can process data from up to four antennas and has additional features like reliability testing and constraints.

MATNAV reads in an option file, which contains all the parameters for processing multiple antennas. Some of the added features of MATNAV are:

- Multiple Antennas (maximum of four antennas can be processed simultaneously)
- Option to apply constraints between the antennas
- Option to test for blunders using reliability methods

The differentially corrected measurements from all the antennas are independently post-processed in a least squares estimation process using carrier smoothing and a cut-off elevation angle of 5º.

If the statistical test option is chosen, a 0.1% significance level for hypothesis Ho and a 10% significance level for hypothesis Ha is used. This means that the probability of rejecting a good observation is 0.001 and the probability of accepting a bad observation is 0.1, which is highly significant (Mikhail and Gracie, 1998). These are some of the optimal significance levels (Leick, 1995) for which the non-centrality parameter is given in Table 1

The standardized least squares residuals are then tested against this threshold. If any of the observations fail the statistical test, subsets of the original set of observations are formed. The statistical test is again performed on each of these subsets. If only one subset passes the test, then the blunder is eliminated, and if none of the subsets pass the statistical test then there is more than one blunder and all the observations are discarded. This is a very conservative approach but if this method is used for real time navigation, then a message can be generated to the user to inform about the presence of an undetectable blunder in the measurement. However, if more than one subset passes the statistical test, then the subset with the smallest sum of squared residuals is chosen for computing the position. The methodology is detailed in the flow chart shown in Figure 2.

In addition to the statistical test, constraints are applied if there are more than two antennas. The standard deviation of the constraints depends on the external method of measuring the baseline. For the experiments conducted, the baseline lengths were measured with a tape and a standard deviation of about 1 cm was chosen.

To study the performance of reliability and constraints the data was processed with/without constraints and with/without reliability for all combinations of antennas.

Latitude (ϕ), longitude (λ), altitude (h) and clock bias (c_b) are the four unknown parameters that are estimated for each antenna using least-squares estimation. If there are four antennas, the total number of estimated parameters is 16. According to the prevailing visibility conditions, the number of parameters that are estimated may vary from 4 to 16. Then depending on the antennas used at a particular instant, appropriate constraints can be applied.

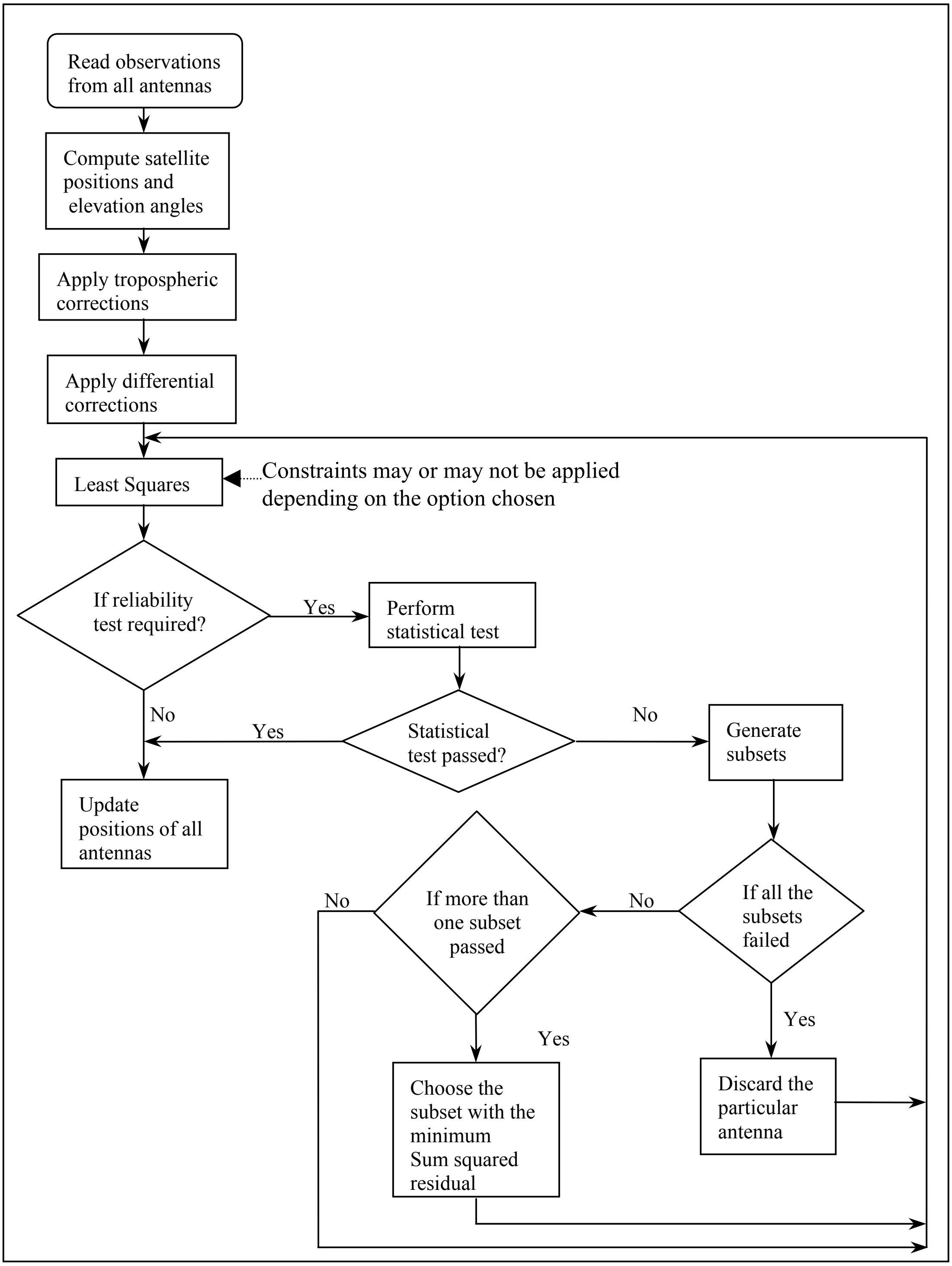

Figure 2: Flow chart of the statistical test

TEST DESCRIPTION

Data was collected on June 24, June 30 and September 9 1999 (herein referred to Days 1,2 and 3), from four GPS antenna/receiver systems mounted on a passenger vehicle and a fifth antenna on the roof of the Engineering Building at the University of Calgary (UofC). The four antennas on the vehicle were connected to four NovAtel MiLLennium™ GPS receivers. A NovAtel Beeline™ GPS receiver was mounted on a pre-surveyed pillar on the roof of the Engineering building acting as a reference station generating differential corrections. Although the MiLLennium™ receivers are dual frequency units (whereas the Beeline™ is single frequency), only the L1 data was used during post-analysis. NovAtel's high performance active antenna (model 501) was used in the reference station and on the vehicle. Raw measurement and ephemeris records were logged from the vehicle and reference station receivers at a rate of 1 Hz.

The location of the antennas on the roof of the vehicle is shown in Figure 3. The antennas were placed at least 0.5 m apart from each other and are designated as Antennas A, B, C and D. Two of the antennas, labeled A and B, were mounted on ski racks and antennas C and D were mounted on magnetic mounts.

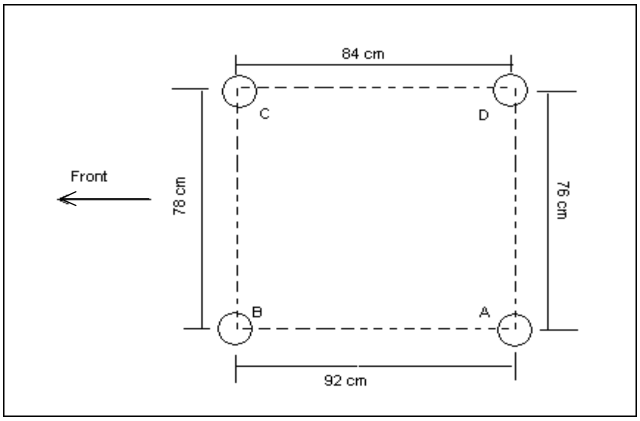

Figure 3: Antenna locations on vehicle roof

Figure 4: Vehicle setup

Data collected on Day 1 was unusable due to a bad power connection on one of the antennas. Hence, data collected on Days 2 and 3 were used for the analysis.

A 30-km route in Calgary was chosen for the test, which encompasses four sections that can be classified as:

- *Section 1 - Open sky:* The route traversed in Section 1 has a very clear view of the sky, which is free from obstructions on both sides of the road. The total distance traveled in this section was 6 km.
- *Section 2 - Dense urban:* Section 2 was chosen in the downtown section of Calgary, specifically to test the performance under a dense urban canyon environment. The total distance traveled in Section 2 was approximately 9 km.
- *Section 3 - Heavy foliage:* This route was chosen in a residential block in south west Calgary. The route was chosen to analyze the performance under dense foliage conditions. The total distance traveled in this section was approximately 8 km.
- *Section 4 - Open sky, semi-urban:* Section 4 was basically a retrace of Section 1 and the total distance covered in this section was approximately 7 km.

A further description of the test environment is given in Nayak et al. (2000a).

RESULTS AND ANALYSIS

The data for each section, along with different antenna combinations are processed in the following modes:

- No constraints, No reliability (NCNR)
- No constraints, With reliability (NCWR)
- With constraints, No reliability (WCNR)
- With constraints, With reliability (WCWR)

The results for each of the four sections of the test were analyzed individually and some of the different scenarios are show below. The position errors (horizontal components) were computed by comparing the MATNAV positions with a highly accurate (< 20 cm) digital map data. The co-ordinates of the map correspond to the center of the street and were generated from photogrammatric techniques. Since vehicle was moving in and out of the center of the road depending on whether the road was a single lane or a two-lane road, this makes it practically impossible to estimate the distance from the center of the road to the vehicle to compute the errors. Hence the results shown below include these errors due to the motion of the vehicle from the center.

Section 1 Results

The RMS of the absolute position errors for Section 1 for data collected on Days 2 and 3 are shown in Figures 5 and 6, respectively. The RMS values are averaged across antennas and are represented for each reliability-constraint scenario. To apply constraints, at least two antennas need to be present, and hence the scenario with one antenna and a constraint is not possible and therefore not included on the graphs. Since Section 1 of the test was in an open area with negligible multipath, the RMS errors do not show much improvement with the application of constraints and reliability. The average RMS error increases from 3.2m (scenario NCNR) to 4.01m (scenario

WCWR). This is because a large error in one antenna can bias the coordinates of the rest of the antennas.

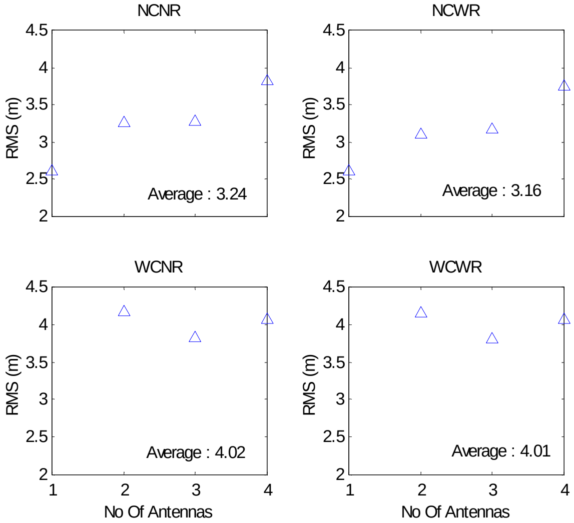

Figure 5: Average RMS agreement between DGPS and map co-ordinates for various scenarios, Section 1, Day 2

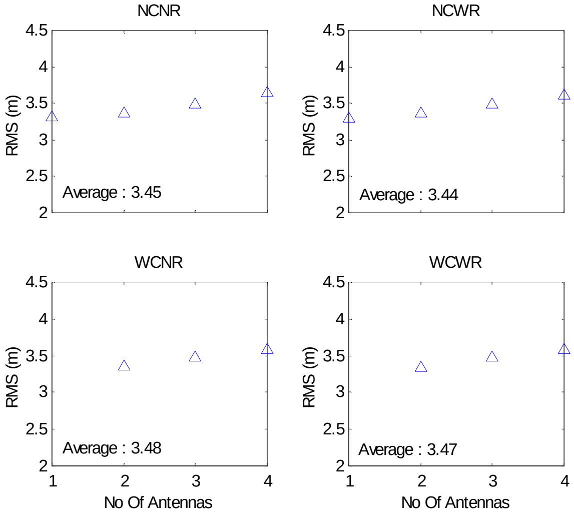

Figure 6: Average RMS agreement between DGPS and map co-ordinates for various scenarios, Section 1, Day 3

The use of a reliability algorithm with constraints showed hardly any improvements in the position domain for Section 1, however the quality of the solution is also not known. Therefore to assess the overall reliability, the external reliability from the minimum detectable blunder is computed as described earlier. The average of these errors for Section 1 on Days 2 and 3 are shown in Figures 7 and 8 respectively.

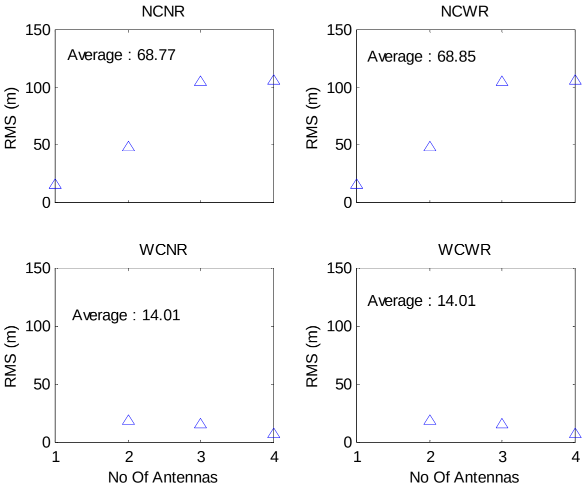

Figure 7 : External reliability RMS errors for various scenarios, Section 1, Day 2

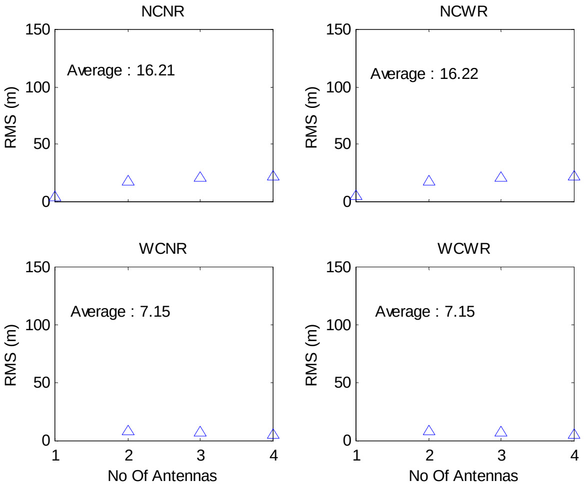

Figure 8: External reliability RMS errors for various scenarios, Section 1, Day 3

On Day 3 the maximum horizontal errors reduce by more than 50% with the application of constraints and more than 80% on Day 2. Also the reliability improves by 30% when four antennas are used instead of two antennas. This indicates that having additional antennas helps to achieve better reliability as opposed to having a single antenna and perform only statistical test. This result is consistent with the improvement in MDB by the addition of extra observations (Salzmann, 1991).

Section 2 Results

The horizontal position errors and the maximum expected horizontal errors (external reliability) for Section 2 (dense urban environment) using data collected on Day 2 is shown in Figures 9 and 10. Table 2 summarizes the RMS errors of the position errors for the various reliability and constraint scenarios for data from Days 2 and 3.

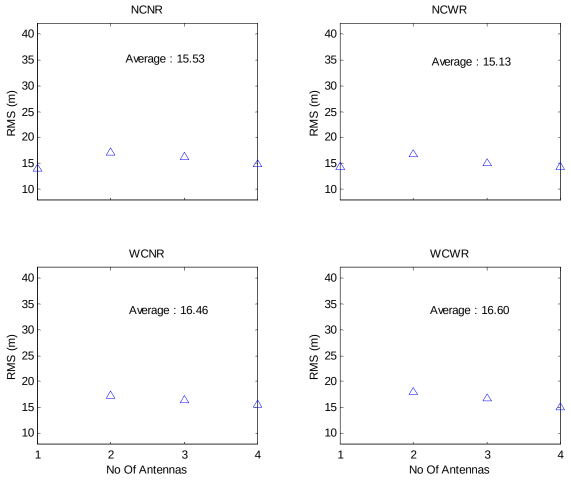

Figure 9 : Average RMS agreement between DGPS and map co-ordinates for various scenarios, Section 2, Day 2

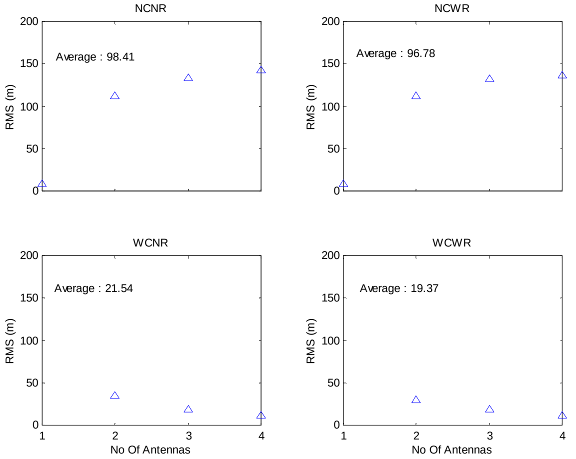

Figure 10: External reliability RMS errors for various scenarios, Section 2, Day 2

There is not much improvement in position error between scenarios NCNR and WCWR on Day 2 however; on Day 3 the improvement is more than 50%. The external reliability from Figure 10 improves from 98m for multiple antennas without constraints and reliability, to 16m with the application of constraints and reliability. Also, from Figure 10 it can be seen that by using additional antennas, better reliability is achieved as opposed to having a single antenna. The expected horizontal errors are in the range of 20m for Section 2 as opposed to 14m in Section 1. This is because the MDB, which decides the external reliability, is a function of the measurement precision as well as geometry (Salzmann, 1991), both of which are poor in the heavy urban environment.

Table 2: Average RMS position errors for Section 2 under various scenarios

Section 2 Day – 2	**NCNR RMS (m)**	**NCWR RMS (m)**	**WCNR RMS (m)**	**WCWR RMS (m)**
1 antenna	13.96	14.34	NA	NA
2 antennas	17.07	16.81	17.36	18.05
3 antennas	16.23	15.07	16.50	16.73
4 antennas	14.87	14.31	15.52	15.03
Section 2 Day – 3				
1 antenna	23.08	23.02	NA	NA
2 antennas	13.43	13.27	11.23	10.33
3 antennas	15.41	15.26	11.07	11.07
4 antennas	34.94	40.79	20.17	16.23

Section 3 Results

The performance of the algorithm in section 3 (dense foliage environment) is shown below. Table 3 summarizes the external reliability for various scenarios.

Table 3: Average RMS errors of the external reliability for Section 3

Section 3 Day – 2	**NCNR RMS (m)**	**NCWR RMS (m)**	**WCNR RMS (m)**	**WCWR RMS (m)**
1 antenna	14.9	14.9	NA	NA
2 antennas	122.6	105.1	28.2	28.3
3 antennas	271.2	206.2	15.7	15.4
4 antennas	249.4	216.2	9.7	9.3
Section 3 Day – 3				
1 antenna	13.7	138.5	NA	NA
2 antennas	107.1	57.5	30.8	29.9
3 antennas	205.3	170.82	22.1	21.9
4 antennas	224.0	200.5	9.7	9.9

A large improvement in the external reliability in Section 3 can be seen from single antenna case to multi-antenna case. Also the reliability improves with the addition of constraints, which agrees with the results from section 1 and section 2.

Overall, a significant improvement in the external reliability and position accuracy is observed with the proposed algorithm. The effectiveness of this technique however depends on the magnitude of multipath blunder present and the ability to detect the same with the proposed statistical test.

From Table 3 it can be seen that by applying constraints large improvements in external reliability can be seen even without conducting the statistical test. This indicates better performance can be expected by just having more than one antenna for navigation. However, multipath blunders in any one of the antennas can bias the position of all the antennas.

Having more antenna/receivers drives up the overall cost of the system. Hence, depending on the reliability needs of the application, choice of number of antenna/receivers that needs to be used should be made. Even with two antennas reasonable performance benefits can be seen from an automobile navigation perspective.

CONCLUSIONS AND FUTURE WORK

The paper focused on the development of an algorithm to detect and reject multipath based on reliability analysis and multiple antennas. Field data was collected to verify the algorithms and to assess their performance under several operational environments.

The algorithm did not substantially improve the position accuracy in an open sky environment (section 1). The blunders or multipath errors in this case are too small to be detected by the statistical test, however by applying constraints the positions of all the four antennas are bounded by each other and results in a better averaging of the positions.

However, the results were more promising in urban (section 2) and foliage sections (section 3). The RMS position errors in urban areas on Day 3 with four antennas, and without any constraints and reliability, is around 20 m, but improves to 13 m by applying constraints and performing the reliability test. However, the improvements on Day 2 are much smaller. This is because the multipath environment is different during different runs and the improvement is proportional to the blunder. This method does not eliminate multipath completely but depends on the detection capability of the statistical test. Some of the blunders were removed with the addition of constraints and reliability tests and sometimes there were more than one blunder in the observation set and the statistical test failed. The average position errors did not improve substantially due to the increase in DOP, which resulted when some measurements were discarded. Also, most often there were not sufficient measurements available to perform a reliability test.

The results show that having a reliability test and constraints together is better than having only reliability or constraints. The number of antenna/receivers that needs to be used depends on the reliability requirement of the application. The reliability increases with every additional antenna. However, for non-critical automobile navigation applications two antennas with reliability and constraints are adequate.

One of the methods to improve MDB is to increase the redundancy. This can be achieved by estimating fewer parameters during estimation, which can be accomplished by having a common clock and solving for one clock parameter for all the receivers instead of estimating clock parameters for individual receiver. The accuracy and robustness of the system can also be improved if the vehicle trajectory information is available from an independent source like an IMU. The results for such a system are discussed in Nayak et al (2000b).

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