

Synergy Between GPS Code, Carrier and Signal-to-Noise-Ratio Multipath Errors

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ABSTRACT

GPS code, carrier and SNR measurements are corrupted by multipath signals that can significantly affect the quality of data used for static and kinematic positioning applications. A tool to simulate code, carrier and signal-to-noise ratio (SNR) multipath errors in a user-defined, multi-antenna and multi-reflector environment for multipath error analysis is developed and described. Various relationships between parameters such as the multipath amplitude, phase and frequency with the satellite dynamics, antenna-reflector distance, antenna-reflector geometry, signal frequency are derived. Multipath spatial and temporal correlation are analyzed using simulated and field data. The similarities and differences of code, carrier and SNR multipath error characteristics are also investigated.

INTRODUCTION

Multipath is one of the most significant sources of error in high accuracy satellite-based applications using differential positioning. Multipath is the phenomenon whereby a signal is reflected or diffracted from various objects in the environment and arrives at the receiver via

multiple paths [1]. It can be as large as several meters for the code and several centimeters for the carrier phase with currently available receiver technologies, and it cannot be removed through differential positioning due to its highly localized nature [2].

Multipath effects on a Pseudo Random Noise (PRN) ranging receiver were studied by Hagerman [3]. His comprehensive investigation of multipath effects on a Delay Lock Loop (DLL) was further extended, by Van Nee [4], Braasch [2], and Ray and Cannon [5], for example. Multipath was experienced by several researchers including Falkenberg et al. [6] and Lachapelle et al. [7] in marine DGPS experiments, and Cannon and Lachapelle [8] in static and dynamic land experiments. Tranquilla and Carr [9] observed multipath occurring at various locations, such as rock embankments, high-tension overhead wires and saltwater/freshwater. Notably, Georgiadou and Kluesberg [10] detected carrier phase multipath using dual frequency receivers. Similarly, there have been numerous publications [1, 11-15] on multipath experiences in various applications.

In this paper code and carrier multipath errors characteristics are analyzed by investigating the impact of multipath parameters from a geometrical perspective through simulation models. The effect of multiple reflected signals on a dot-product type of non-coherent delay lock loop discriminator is also analyzed. Furthermore, the code and carrier multipath spatial and temporal correlation and similarities and differences among code, carrier and SNR multipath errors are investigated using simulation and field data. These analyses will help in assessing multipath errors in a given environment and develop multipath mitigation techniques.

CODE MULTIPATH IN A DOT-PRODUCT DISCRIMINATOR

Figure 1 shows a block diagram of a typical GPS receiver tracking loop, which consists of a DLL for code tracking and a Costas Loop for carrier tracking [16, 17]. In practice, the DLL in a GPS receiver generally has a non-coherent type of discriminator [18, 19]. An n parallel channel receiver will have n such sets of blocks corresponding to each independent tracking loop.

In a receiver, the digitized IF signal is input to each of these parallel channels. The input signal is beat with the locally generated in-phase and quadrature-phase replicas of the carrier. The signal is then correlated with the prompt (P), early (E) and late (L) versions of the locally generated code, and the correlation values are integrated for a pre-detection integration period. The early and late correlation values in the in-phase (I) and quadrature-phase (Q) arms (IE, IL, QE, QL) are generally used for code tracking, whereas the prompt correlation values (IP, QP) are used for carrier tracking. Some code discriminators, such as the dot-product type, use prompt correlation values as well.

The incoming GPS satellite signal in a receiver consists of a direct signal and, often, more than one reflected signal. Each of these direct and reflected signals consists of a carrier modulated by the code as well as the navigation data bits. Data bits are extracted in the receiver at a later stage and are of no concern as long as the pre-detection integration period in the receiver tracking loops are from one data bit boundary to another. The composite input signal, neglecting the navigation data bit and assuming that the multipath signal frequency is the same as the direct signal frequency, can be expressed by the following equation:

$$s_I(t) = A \sum_{i=0}^n \alpha_i c(t - \tau_i) \cos(\omega_0 t + \gamma_i) \quad (1)$$

where,

A is the satellite signal carrier amplitude (volt)

n is the number of direct and reflected signals

α_i are the direct and reflected signal coefficient, where α_0 corresponds to the direct signal and is equal to 1

$c(t)$ is the GPS C/A or P code

τ_i is the satellite signal code delay, where τ_0 corresponds to the direct signal code delay (s)

ω_0 is the satellite signal carrier frequency (rad/s), and

γ_i is the satellite signal carrier phase, where γ_0 corresponds to the direct signal phase (rad).

The reflected signal carrier phase differs from the direct signal carrier phase by i) the differential path delay, and ii) the change in the phase during reflection. The former of these factors depends upon the location of the reflector with respect to the antenna, and the latter is a function of the reflector's physical properties and the angle of incidence as described in the previous section.

The local replica of the carrier has frequency and phase equal to the receiver's estimate of the incoming satellite signal frequency and phase. Similarly, the locally-generated prompt code has a delay equal to the receiver's estimate of the incoming signal code delay. The locally generated signal, combining code and carrier, in the in-phase arm for the prompt correlator then can be expressed as:

$$s_{IP}(t) = c(t - \hat{\tau}_0) \cos(\hat{\omega}_0 t + \hat{\gamma}_0) \quad (2)$$

where,

$\hat{\tau}_0$ is the receiver's estimate of the direct signal code delay (s)

$\hat{\omega}_0$ is the receiver's estimate of the signal carrier frequency (rad/s), and

$\hat{\gamma}_0$ is the receiver's estimate of the signal carrier phase (rad).

Then, the in-phase prompt correlation value (IP), assuming that the incoming and the locally generated carrier frequencies are the same, is given by:

$$\text{IP} = \overline{s_i(t)s_{\text{IP}}(t)} = \sum_0^n \alpha_i \frac{A}{2} R(\hat{\tau}_c - \tau_i) \cos(\gamma_i - \hat{\gamma}_c) \quad (3)$$

where,

$R()$ is the correlation function. For a PRN code without band limitation it is defined as,

$$A(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, & |\tau| \leq T \\ 0, & |\tau| > T \end{cases} \quad (4)$$

T is the PRN code bit period (s)

$\hat{\tau}_c$ is the receiver estimate of the incoming signal code delay (m), and

$\hat{\gamma}_c$ is the receiver estimate of the incoming signal carrier phase (rad).

Similarly, the in-phase early (IE), in-phase late (IL), quadrature-phase prompt (QP), quadrature-phase early (QE), and quadrature-phase late (QL) correlation values in the presence of multipath are, respectively,

$$\begin{aligned}
IE &= \sum_0^n \alpha_i \frac{A}{2} R(\hat{\tau}_c - \tau_i + T_d) \cos(\gamma_i - \hat{\gamma}_c) \\
IL &= \sum_0^n \alpha_i \frac{A}{2} R(\hat{\tau}_c - \tau_i - T_d) \cos(\gamma_i - \hat{\gamma}_c) \\
QP &= \sum_0^n \alpha_i \frac{A}{2} R(\hat{\tau}_c - \tau_i) \sin(\gamma_i - \hat{\gamma}_c) \\
QE &= \sum_0^n \alpha_i \frac{A}{2} R(\hat{\tau}_c - \tau_i + T_d) \sin(\gamma_i - \hat{\gamma}_c) \\
QL &= \sum_0^n \alpha_i \frac{A}{2} R(\hat{\tau}_c - \tau_i - T_d) \sin(\gamma_i - \hat{\gamma}_c)
\end{aligned} \tag{5-9}$$

For a non-coherent dot-product discriminator, the discriminator function is given by [16],

$$D_n = IP(IE - IL) + QP(QE - QL) \tag{10}$$

In the presence of a single dominant reflector, Equation 10 can be transformed by using expressions from Equations 3 to 9, to give,

$$\begin{aligned}
D_{nm} &= \sum_{i=0}^1 \alpha_i R(\hat{\tau}_c - \tau_i) \cos(\gamma_i - \hat{\gamma}_c) \left\{ \begin{array}{l} \sum_{i=0}^1 \alpha_i R(\hat{\tau}_c - \tau_i + T_d) \cos(\gamma_i - \hat{\gamma}_c) \\ - \sum_{i=0}^1 \alpha_i R(\hat{\tau}_c - \tau_i - T_d) \cos(\gamma_i - \hat{\gamma}_c) \end{array} \right\} \\
&\quad + \sum_{i=0}^1 \alpha_i R(\hat{\tau}_c - \tau_i) \sin(\gamma_i - \hat{\gamma}_c) \left\{ \begin{array}{l} \sum_{i=0}^1 \alpha_i R(\hat{\tau}_c - \tau_i + T_d) \sin(\gamma_i - \hat{\gamma}_c) \\ - \sum_{i=0}^1 \alpha_i R(\hat{\tau}_c - \tau_i - T_d) \sin(\gamma_i - \hat{\gamma}_c) \end{array} \right\}
\end{aligned} \tag{11}$$

Equation 11 can be expanded and simplified to obtain the following expression:

$$\begin{aligned}
D_{nm} &= R(\hat{\tau}_c - \tau_0) \{ R(\hat{\tau}_c - \tau_0 + T_d) - R(\hat{\tau}_c - \tau_0 - T_d) \} + \\
&\quad \alpha_1^2 R(\hat{\tau}_c - \tau_1) \{ R(\hat{\tau}_c - \tau_1 + T_d) - R(\hat{\tau}_c - \tau_1 - T_d) \} + \\
&\quad \left\{ \alpha_1 R(\hat{\tau}_c - \tau_0) [R(\hat{\tau}_c - \tau_1 + T_d) - R(\hat{\tau}_c - \tau_1 - T_d)] + \right. \\
&\quad \left. \alpha_1 R(\hat{\tau}_c - \tau_1) [R(\hat{\tau}_c - \tau_0 + T_d) - R(\hat{\tau}_c - \tau_0 - T_d)] \right\} \cos(\gamma_0 - \gamma_1)
\end{aligned} \tag{12}$$

For continuous tracking, D_{nm} is equated to zero and the resultant delay error is computed. Equation 12 does not have the term $\hat{\gamma}_c$, which appears in the case of a coherent discriminator function. That means that in this case, (or for that matter, in any non-coherent discriminator), code tracking does not depend upon the carrier phase tracking, as long as the carrier frequency is locked. The multipath error can be computed by assuming that $\tau_0 = 0$; in that case $\hat{\tau}_c$ is the multipath error.

In the case of multiple reflectors, the upper limit of the summation in Equation 11 will be equal to the number of reflectors in the environment. Assuming three reflectors in the environment, Equation 12 may be expanded and rearranged to give the following expression:

$$\begin{aligned}
D_{nm} = & R(\hat{\tau}_c - \tau_0)\{R(\hat{\tau}_c - \tau_0 + T_d) - R(\hat{\tau}_c - \tau_0 - T_d)\} + \\
& \alpha_1^2 R(\hat{\tau}_c - \tau_1)\{R(\hat{\tau}_c - \tau_1 + T_d) - R(\hat{\tau}_c - \tau_1 - T_d)\} + \\
& \alpha_2^2 R(\hat{\tau}_c - \tau_2)\{R(\hat{\tau}_c - \tau_2 + T_d) - R(\hat{\tau}_c - \tau_2 - T_d)\} + \\
& \alpha_3^2 R(\hat{\tau}_c - \tau_3)\{R(\hat{\tau}_c - \tau_3 + T_d) - R(\hat{\tau}_c - \tau_3 - T_d)\} + \\
& \alpha_1 R(\hat{\tau}_c - \tau_0)\{R(\hat{\tau}_c - \tau_1 + T_d) - R(\hat{\tau}_c - \tau_1 - T_d)\}\cos(\gamma_0 - \gamma_1) + \\
& \alpha_2 R(\hat{\tau}_c - \tau_0)\{R(\hat{\tau}_c - \tau_2 + T_d) - R(\hat{\tau}_c - \tau_2 - T_d)\}\cos(\gamma_0 - \gamma_2) + \\
& \alpha_3 R(\hat{\tau}_c - \tau_0)\{R(\hat{\tau}_c - \tau_3 + T_d) - R(\hat{\tau}_c - \tau_3 - T_d)\}\cos(\gamma_0 - \gamma_3) + \\
& \alpha_1 R(\hat{\tau}_c - \tau_1)\{R(\hat{\tau}_c - \tau_0 + T_d) - R(\hat{\tau}_c - \tau_0 - T_d)\}\cos(\gamma_0 - \gamma_1) + \\
& \alpha_2 R(\hat{\tau}_c - \tau_2)\{R(\hat{\tau}_c - \tau_0 + T_d) - R(\hat{\tau}_c - \tau_0 - T_d)\}\cos(\gamma_0 - \gamma_2) + \\
& \alpha_3 R(\hat{\tau}_c - \tau_3)\{R(\hat{\tau}_c - \tau_0 + T_d) - R(\hat{\tau}_c - \tau_0 - T_d)\}\cos(\gamma_0 - \gamma_3) + \\
& \alpha_1 \alpha_2 R(\hat{\tau}_c - \tau_1)\{R(\hat{\tau}_c - \tau_2 + T_d) - R(\hat{\tau}_c - \tau_2 - T_d)\}\cos(\gamma_1 - \gamma_2) + \\
& \alpha_1 \alpha_3 R(\hat{\tau}_c - \tau_1)\{R(\hat{\tau}_c - \tau_3 + T_d) - R(\hat{\tau}_c - \tau_3 - T_d)\}\cos(\gamma_1 - \gamma_3) + \\
& \alpha_1 \alpha_2 R(\hat{\tau}_c - \tau_2)\{R(\hat{\tau}_c - \tau_1 + T_d) - R(\hat{\tau}_c - \tau_1 - T_d)\}\cos(\gamma_1 - \gamma_2) + \\
& \alpha_1 \alpha_3 R(\hat{\tau}_c - \tau_3)\{R(\hat{\tau}_c - \tau_1 + T_d) - R(\hat{\tau}_c - \tau_1 - T_d)\}\cos(\gamma_1 - \gamma_3) + \\
& \alpha_2 \alpha_3 R(\hat{\tau}_c - \tau_2)\{R(\hat{\tau}_c - \tau_3 + T_d) - R(\hat{\tau}_c - \tau_3 - T_d)\}\cos(\gamma_2 - \gamma_3) + \\
& \alpha_2 \alpha_3 R(\hat{\tau}_c - \tau_3)\{R(\hat{\tau}_c - \tau_2 + T_d) - R(\hat{\tau}_c - \tau_2 - T_d)\}\cos(\gamma_2 - \gamma_3)
\end{aligned} \tag{13}$$

Equation 13 is used for simulating the code multipath error due to three reflectors in the environment.

CARRIER PHASE MULTIPATH

In a GPS receiver, the carrier phase is measured by accumulating the phase of the locally generated carrier. In the absence of multipath, the local carrier locks onto the direct carrier very accurately, and as a result, the true phase difference between the incoming signal carrier and the locally-generated carrier is nearly zero, (actually zero mean), at steady state. In the presence of multipath, however, the composite signal phase shifts from the direct signal phase, and the NCO-generated local carrier locks onto the composite carrier phase, resulting in an error in the phase measurement. This error is equal to the difference between the composite signal carrier phase and the direct signal carrier phase.

Using Equations 3 to 9, and assuming that there is a single dominant reflector and the local carrier frequency is the same as the incoming carrier frequency, the arctan discriminator function can be expressed as [16],

$$\begin{aligned}
 D_r &= \arctan\left(\frac{QP}{IP}\right) \\
 &= \arctan\left(\frac{R(\hat{\tau}_c - \tau_0)\sin(\gamma_0 - \hat{\gamma}_c) + \alpha_1 R(\hat{\tau}_c - \tau_1)\sin(\gamma_1 - \hat{\gamma}_c)}{R(\hat{\tau}_c - \tau_0)\cos(\gamma_0 - \hat{\gamma}_c) + \alpha_1 R(\hat{\tau}_c - \tau_1)\cos(\gamma_1 - \hat{\gamma}_c)}\right)
 \end{aligned} \tag{14}$$

The carrier tracking loop tries to minimize D_r during signal tracking, and generally its value will be close to zero (actually zero mean). Assuming τ_0 and γ_0 to be zero, replacing $\Delta\Psi = \hat{\gamma}_c - \gamma_0$,

equating Equation 14 to zero, and by performing the proper manipulation, the following expression is obtained:

$$\Delta\Psi = \arctan\left(\frac{\alpha_1 R(\hat{\tau}_c - \tau_1) \sin \gamma_1}{R(\hat{\tau}_c) + \alpha_1 R(\hat{\tau}_c - \tau_1) \cos \gamma_1}\right) \quad (15)$$

Here, $\Delta\Psi$ is the difference between the composite signal phase, (which is tracked by the receiver), and the direct signal phase; it is therefore the carrier phase multipath error. From Equation 15, it is clear that the reflection coefficient, multipath delay and the multipath phase are the multipath parameters. These multipath parameters are always defined with respect to the direct signal.

Following the same procedure, the carrier phase multipath due to multiple (in this case 3) reflectors is given by:

$$\Delta\Psi = \arctan\left\{\frac{\alpha_1 R(\hat{\tau}_c - \tau_1) \sin \gamma_1 + \alpha_2 R(\hat{\tau}_c - \tau_2) \sin \gamma_2 + \alpha_3 R(\hat{\tau}_c - \tau_3) \sin \gamma_3}{R(\hat{\tau}_c) + \alpha_1 R(\hat{\tau}_c - \tau_1) \cos \gamma_1 + \alpha_2 R(\hat{\tau}_c - \tau_2) \cos \gamma_2 + \alpha_3 R(\hat{\tau}_c - \tau_3) \cos \gamma_3}\right\} \quad (16)$$

This expression is used to simulate carrier phase multipath errors due to multiple reflectors.

In the case of a single reflector, the multipath error reaches an absolute maximum when the reflected signal phase is perpendicular to the composite signal phase. Then the maximum value is then given by:

$$\Delta\Psi = \arcsin\left(\frac{\alpha_1 R(\hat{\tau}_c - \tau_1)}{R(\hat{\tau}_c)}\right) \quad (17)$$

The reflected signal phases corresponding to the maxima and minima of the error can be computed by differentiating Equation 15 with respect to γ_1 , equating it to zero, and solving for γ_1 . By performing the steps described above it can be determined that the multipath errors reach extreme values (maxima and minima) at:

$$\gamma_1(\text{max}) = \cos^{-1}\left(-\frac{\alpha_1 R(\hat{\tau}_c - \tau_1)}{R(\hat{\tau}_c)}\right) \quad (18)$$

and

$$\gamma_1(\text{min}) = 2\pi - \cos^{-1}\left(-\frac{\alpha_1 R(\hat{\tau}_c - \tau_1)}{R(\hat{\tau}_c)}\right). \quad (19)$$

From Equations 18 and 19, it is evident that for weak or long delay reflectors, the maxima and minima take place when the multipath phase is close to 90 and 270 degrees. However, for strong and close-by reflectors, the maxima and minima occur at close to 180 degrees of the multipath phase.

Unlike code multipath errors, which are largely affected by the pre-detection bandwidth, carrier multipath errors are not greatly affected by the bandwidth limitation. Lowering the bandwidth has two effects: i) the code multipath errors depend on the bandwidth and, in turn, affect the

carrier multipath errors to a small extent, and ii) the change in shape of the prompt correlation triangle affects the carrier multipath errors to a small extent.

SNR MULTIPATH ERROR

Multipath affects not only the code range and carrier phase measurements, but also the measured signal power, which is an average of the composite signal power due to the direct and reflected signal carrier. As the reflected signal relative phase varies with time, the power of the composite signal also varies with time, and so does the measured power.

It should be emphasized that the code and data bits in the GPS signal do not contribute to the signal power, as they merely change the phase of the carrier depending upon the modulation technique employed. The signal power with or without the data and code bits remains the same. Therefore, the receiver determines the power of the carrier, not the code and data.

In a receiver, the average signal power is generally measured using the prompt correlators and is given by,

$$P = IP^2 + IQ^2 \quad (20)$$

By assuming a uniform antenna gain pattern and a single dominant reflector, and by replacing the values of IP and IQ from Equations 3 to 9, the signal power can be found and is given by,

$$\begin{aligned} P &= R^2(\hat{\tau}_c) + \alpha_1^2 R^2(\hat{\tau}_c - \tau_1) + 2\alpha_1 R(\hat{\tau}_c) R(\hat{\tau}_c - \tau_1) \cos \gamma_1 \\ &= R^2(\hat{\tau}_c) \left(1 + \alpha_1^2 \alpha'^2 + 2\alpha_1 \alpha' \cos \gamma_1 \right) \end{aligned} \quad (21)$$

where the correlation ratio is, $\alpha' = \frac{R(\hat{\tau}_c - \tau_1)}{R(\hat{\tau}_c)}$. (22)

From Equation 21, the average signal power in the receiver is a function of the reflection coefficient, multipath delay and multipath phase. This equation is used for SNR multipath simulation.

MULTIPATH FROM A GEOMETRICAL PERSPECTIVE

In Figure 2, a typical multipath scenario is shown, whereby A0 to A4 are several antennas placed close together in a multi-antenna system, and the reflections from two sources to A0 are shown. The other four antennas will also be affected by the reflected signals in a similar way.

In the diagram, θ and φ are the elevation and azimuth of the direct signal to the antenna, while θ_k and φ_k are the elevation and azimuth of the k th reflected signal to the antenna. The distance between the antenna and the reflector in the horizontal plane is denoted by d_k , where, k represents a particular reflector.

Two distinct scenarios are shown in the figure. In the first case, (Reflector 1), the antenna (A0) is closer to the satellite compared to the reflector, whereas in the second case, (Reflector 2), the reflector is closer to the satellite compared to the antenna (A0). These two cases are generalized situations and representative of all the possible scenarios of the antenna-reflector geometry.

Since the satellite is approximately 20,000 km above the earth, the GPS signal can be assumed to travel as parallel rays to the earth's surface. A plane wavefront perpendicular to the line of sight

can be assumed to have the same carrier phase. When this plane intersects the phase center of Antenna 0, it has the same carrier phase at all points on it, including point P_1 (which is the intersection of the plane and the line of sight from Reflector 1 to the satellite). Therefore, the differential path delay of this reflected signal with respect to the direct signal is $\overline{P_1R_1} + \overline{R_1O}$. The corresponding differential phase delay is computed by dividing the differential path delay by the signal wavelength. This assumes no phase change due to reflection of the signal. This assumption is acceptable to characterize multipath errors and their dependency on geometry in a relative sense. To determine the absolute multipath errors, however, the phase change due to reflection of the signal should be accounted for.

Similarly, for case 2, a plane perpendicular to the line-of-sight from Reflector 2 to the satellite intersects the line-of-sight from the antenna under consideration at point P_2 . In this case, the differential path delay is given by $\overline{R_2O} - \overline{P_2O}$.

Therefore, if the direct signal phase at the antenna is available, the reflected signal phase can be computed by adding the differential phase delay due to the differential path delay (under the above mentioned assumption), to the direct signal phase.

In order to compute the effects of multipath, the above mentioned differential path delays need to be formulated by a mathematical expression. Using solid geometry, it can be shown that the differential path delay in either situation is given by (see Appendix A),

$$a_k = d_k \left(\frac{1}{\cos \theta_k} - \tan \theta_k \sin \theta - \cos \theta \cos(\varphi - \varphi_k) \right) \quad (23)$$

where,

a_k is the differential path delay of the k th reflected signal (m)

d_k is the horizontal distance between the antenna and the k th reflector (m)

θ is the elevation of the direct satellite signal (rad)

φ is the azimuth of the direct satellite signal (rad)

θ_k is the elevation of the k th reflected signal (rad), and

φ_k is the azimuth of the k th reflected signal (rad)

The differential path delay expression is a function of the satellite elevation and azimuth, the reflected signal elevation and azimuth, and the antenna-reflector distance in the local level horizontal plane. This expression is further exploited to analyze the behavior of the carrier phase multipath error.

With the assumption that the multipath phase is only due to the differential path delay, it can be expressed as,

$$\gamma_{0k} = \gamma_k - \gamma_0 = \frac{2\pi a_k}{\lambda_L} \quad (24)$$

where

γ_0 is the direct signal phase at the antenna phase center (rad)

γ_k is the k th reflected signal phase at the antenna phase center (rad), and

λ_L is the wavelength of the carrier (m).

The multipath error is directly related to the multipath phase. The multipath error variation is due to the variation in the multipath phase or the differential path delay. The multipath frequency depends upon the rate of change of the multipath phase, or the differential phase delay. The multipath frequency due to a single dominant reflector may be computed by taking the time derivative of the multipath phase expression from Equations 23 and 24, and is given by,

$$\frac{\delta\gamma_{01}}{\delta t} = \frac{2\pi d_1}{\lambda_L} \left(\left\{ \sin \theta \cos(\varphi - \varphi_1) - \cos \theta \tan \theta_1 \right\} \frac{\delta\theta}{\delta t} + \left\{ \cos \theta \sin(\varphi - \varphi_1) \right\} \frac{\delta\varphi}{\delta t} \right) \quad (25)$$

Equation 25 relates the multipath error frequency with satellite dynamics. The expression is obtained under the assumption that the antenna-reflector geometry (defined by d_1 , θ_1 and φ_1) does not change significantly over the period under consideration. This assumption does not generally hold for kinematic receivers, where the antenna-reflector geometry may change rapidly. Furthermore, in stationary situations, the antenna-reflector geometry changes can be taken care of by taking the partial derivatives with respect to the reflected signal elevation and azimuth in Equation 23.

It is evident from Equation 25 that the multipath error frequency is,

- directly proportional to the distance between the antenna and the reflector
- inversely proportional to the wavelength of the carrier signal
- directly proportional to the rate of change of elevation of the satellite
- directly proportional to the rate of change of azimuth of the satellite, and
- dependent upon the antenna-reflector and the line-of-sight vectors.

SIMULATION DESCRIPTION

A Multipath Simulation and Mitigation software (MultiSiM) for the GPS was developed using 'C' programming language on a PC platform. The software consists of two main parts, namely simulation and mitigation. The first part allows the user to define the multipath environment and the antenna setup through the input parameters. The second part, on the other hand, uses various multipath mitigation schemes to reduce the simulated multipath errors.

The major inputs to the simulator are reflector parameters, and antenna parameters. The major outputs from the simulator are true range and phase, measured range and phase contaminated with multipath and receiver noise, and estimated range and phase.

The user can input the number of reflectors per satellite and their locations with respect to the antenna position in order to simulate a controlled multipath environment. The user can also configure the antenna setup, (i.e. the number of antennas in the antenna array), absolute position of one of the antennas (the reference antenna) and relative positions of all other antennas (secondary antennas) with respect to the reference antenna.

The range and phase of the direct and reflected signals at each antenna may be determined by computing the distance traveled by the signal up to the antenna. For the direct signal, it is the distance between the satellite and the antenna while for the reflected signals, it is the total distance from the satellite to the reflector, plus the reflector to the antenna. The phases of the direct and reflected signals are assumed to be only a function of the ranges, and are computed

directly from the ranges. The possible change in phase due to reflection of the signal is not considered in this simulation. The satellite position is determined from stored ephemeris data.

The noiseless measured code range is the sum of the direct range between the antenna and the satellite, and the code multipath error. The code multipath error is computed by using Equation 13 and finding the difference between the zero crossings of the multipath-corrupted discriminator function and the multipath-free discriminator function. A single observation from the direct and the numerous reflected signals is generated per satellite-antenna combination.

The measured carrier phase without noise contains two parts: the integer and fractional cycle components. Assuming that the direct signal is stronger than the indirect signal, the integer cycles in the measured carrier phase are the same as the direct signal's integer cycles. The phase of the fractional cycle of the reflected signal is what actually corrupts the phase of the fractional cycle of the direct signal, depending upon its relative strength and phase. Equation 16 is used to compute the multipath error on the fractional part of the carrier phase.

SIMULATION AND FIELD RESULTS

Multipath Error vs. Reflector Location

Figures 3a to 3c show code multipath errors due to one, two and three reflectors, respectively. For the case of a single reflector at a distance of 15 m (multipath delay 3.6 m) from the antenna, and a reflection coefficient of 0.5, the multipath error is uniform and has slow variations in periodicity due to satellite dynamics (Figure 3a). The reflector and the antenna positions were

assumed to be stationary during the simulation. In the presence of a second reflector at a distance of 50 m (multipath delay 55.2 m) from the antenna, and reflection coefficient of 0.2, the multipath error becomes quite irregular (Figure 3b). The effect of the second reflector is quite significant in spite of a lower reflection coefficient. This is because, from the code multipath error envelope, reflection from a close-by object produces small multipath errors, and therefore less dominant compared to multipath due to a further object [2]. The addition of a third reflector at a distance of 20 m (multipath delay 30.1 m) from the antenna, and a reflection coefficient of 0.2, have made the multipath error more irregular (Figure 3c).

Figures 4a to 4c show carrier phase multipath errors due to one, two and three reflectors respectively as described in the previous section (corresponding to Figures 3a, 3b and 3c). For the case of a single reflector, the multipath error is regular and has a slow variation in periodicity (Figure 4a). In the presence of a second reflector, the multipath error pattern does not change significantly compared to the code multipath case (Figure 4b). This is also evident in the presence of a third reflector (Figure 4c). The reason for the second and third reflectors having a less significant contribution in the carrier phase case can be explained from the carrier phase multipath error envelope [2, 4]. Unlike the code, the carrier phase multipath error amplitude proportionately decreases as the multipath delay increases. Multipath delays in the cases of second and third reflectors are longer than the case of the first reflector. Furthermore, the reflection coefficient of the first reflector is higher than the other two, which makes the first reflector the dominant reflector in this case.

Multipath Spatial Correlation

Figures 5a to 5e show simulated code multipath errors due to three reflectors at five closely-spaced antennas which are placed 5-10 cm from each other. It is assumed that the size of the reflector is much higher compared to the largest distance among the antennas, such that all the antennas in the multi-antenna assembly are affected by the same reflector. It is clear from the figure that the multipath errors are highly correlated between antennas, and have very similar patterns. However, they have different phases due to different differential path delays (hence different multipath phases) of the reflected signal. Due to these phase differences, multipath errors do not get cancelled by taking a single difference of the errors in two closely-spaced antennas.

This is further confirmed using field data. The data was collected on the roof of the Engineering Building of the University of Calgary using NovAtel BeeLine™ receivers [20]. It is a moderate multipath environment with reflections coming from the concrete roof and four surrounding walls. Figures 6a to 6e show code multipath errors for satellite 31 from data collected on October 20, 1998 on five closely spaced antennas separated by 5 to 10 cm. Multipath errors for the antennas are generally consistent, however, unlike simulated multipath errors, the errors in each antenna are not due to the same set of reflectors. Furthermore, the multipath phase changes are not smooth due to the irregular and complex environment compared to the simulated environment. These are the primary reasons for the dissimilarities in the multipath patterns in the closely-spaced antennas. The multipath errors in the antennas do not cancel out through single differencing between the errors in two antennas, which is demonstrated in Figures 7a to 7d. However, their relationships can be exploited to estimate the multipath error at individual antennas as described in Ray et al. [21].

Similar to the code multipath error case, Figures 8a to 8e show simulated carrier multipath errors at the same closely-spaced antennas. It is clear from the figures that the multipath errors are highly correlated. However, they do not cancel out by single differencing across two antennas. This is reconfirmed using field data. Figures 9a to 9e show carrier multipath errors for satellite 31 from the same field data described previously. Figures 10a to 10d show the single differences of the carrier phase multipath errors between the antennas. It is evident from the figures that multipath errors are not the same even for two antennas separated by a small distance. The relationship between multipath errors in closely-spaced antennas can be used to estimate the carrier phase multipath errors in each antenna as described in [22, 23].

Multipath Repeatability and Temporal Correlation

Multipath repeatability and temporal correlation was examined for the code and carrier cases. The data collected on the roof of the Engineering Building as described earlier was used for the investigation. The code multipath error was extracted for satellite 31 (elevation angle 21° - 34°) from data collected on October 7 and 8, 1998, and is shown in Figure 11 using a shaded dark line and shaded light line for the two days. Multipath errors extracted from the data on October 8 are plotted and are offset by approximately four minutes with respect to the errors extracted from the data collected on October 7. As the sidereal day is approximately four less than 24 hours and the antenna-satellite and antenna-reflector geometry repeat exactly after a sidereal day, the multipath errors should repeat after a sidereal day. From the figure, it is clear that multipath errors repeat to a great extent after a sidereal day.

Figure 12 shows the cross-correlation function of the code multipath errors on October 7 and October 8 for satellite 31. From the figure it can be seen that the multipath errors have maximum correlation after a sidereal day. The extent of the repeatability is approximately 90% for satellite 31. A correlation time of approximately 2-3 min was found in this case.

The carrier phase multipath errors for satellite 31 were computed using data collected on October 7, 1998, and are shown in Figure 13 using a shaded dark line. Similarly, the multipath errors for the same satellite was computed using October 8 data, and is superimposed on the same figure but shifted by approximately four minutes in the time (shown using a shaded light line). From the figure, it is clear that the carrier phase multipath errors repeat after a sidereal day.

Figure 14 shows the cross-correlation function of the carrier phase multipath errors on October 7 and October 8 for satellite 31. From the figure it can be seen that the multipath errors have a maximum similarity after a sidereal day. The extent of the repeatability is approximately 70% in this case. A correlation time of approximately 5-6 minutes was found in this case.

Comparing the code and carrier multipath errors in Figures 12 and 14, and Figures 3c and 4c several comments can be made.

i) Code multipath errors have higher frequency components compared to the carrier phase. This is due to the fact that the code and carrier discriminator functions in the receiver tracking loops respond differently in the presence of multipath signals. The code discriminator produces multipath errors of high magnitude due to distant reflectors, which causes high frequency

multipath. Therefore, the code multipath error is dominated by high frequency components. On the other hand, the carrier discriminator produces multipath errors of high magnitude due to close-by reflectors, which cause low frequency multipath errors. From Figures 11 (or 3c) and 13 (or 4c), it can be seen that the dominant reflectors in the code and carrier multipath errors are not the same. For code multipath errors, it is the far away reflectors, whereas for carrier multipath errors, it is the close-by reflectors that dominate the composite multipath errors.

ii) The correlation coefficients (a measure of the day-to-day repeatability) for code multipath errors are higher than those of the carrier phase. This is because the carrier phase residuals have high phase noise, as they are double differenced residuals. Therefore, the effects of the carrier noise are higher compared to that of the code noise on the correlation coefficient. As the receiver noise on day one is uncorrelated with the noise on day two, code residuals, which are comparatively less affected by the code noise, have higher correlation coefficients compared to their carrier phase counterparts.

Multipath Error vs Carrier Frequency

Figure 15 shows simulated multipath errors in L1 and L2 carriers due to three reflectors. In these figures, the dark shaded errors correspond to the L1 carrier and the light shaded errors to the L2 carrier. Several important observations can be made from the figures: i) the multipath error has the same amplitude (in radians) for the L1 and L2 carrier (although when multiplied by the wavelength to convert the error into units of distance, the L2 multipath error has a larger amplitude than that of L1), ii) the multipath error has a different phase for the L1 and L2 carrier. At a particular instant, the multipath error for the L1 and L2 carriers look arbitrary, but over a

time-span, it becomes evident that the error signals have similar patterns. The multipath error dependency on frequency is also explained in Georgiadou and Kleusberg [10].

Code, Carrier and SNR Multipath Error Synergy

The code, carrier and SNR are affected by multipath in different ways. Figure 16 shows the code, carrier and SNR error patterns in the presence of a multipath signal with a reflection coefficient of 0.7. It can be seen that the code and SNR error patterns are in-phase with respect to each other, whereas the carrier phase error pattern is quadrature-phase with respect to the code and SNR errors. The uniform pattern of these errors and their inter-relationships is such that if any of these three errors is known, it may be possible to estimate the other two if a suitable relationship can be established among the three.

Figures 17 and 18 show the code, carrier and SNR multipath errors for satellites 31 and 9 respectively from the data collected on October 20, 1998. It is clear from the figures that the code, carrier and SNR multipath errors have similar patterns. Generally the code multipath has more prominent oscillations compared to the carrier and SNR errors. This suggests that there is one or more far away reflectors, which play a dominant role in the case of the code. The carrier phase multipath looks noisier as it was computed by taking double differences between antennas and between satellites, which increases the amount of thermal noise.

The most interesting observation from Figures 17 and 18 are that the code and SNR multipath errors are of same phase for both satellites. However, the carrier multipath is phase offset with respect to the code and SNR errors. If the carrier multipath is left shifted in time by

approximately 2 minutes, then there is a higher degree of temporal correlation among the code, carrier and SNR errors. This is in accord with the theoretical and simulated relationships among the code, carrier and SNR multipath errors.

CONCLUSIONS

Multipath is a major source of error for high accuracy differential code and carrier positioning. Effective multipath mitigation techniques or multipath avoidance requires a sound understanding of its characteristics. In this paper, various code, carrier and SNR multipath characteristics are analyzed using simulation models and field data.

This paper derives various relationships between multipath parameters such as the multipath amplitude, phase and frequency with respect to satellite dynamics, antenna-reflector distances, antenna-reflector geometry. It was found that presence of multiple reflectors in the environment makes the multipath errors irregular in nature. The location of the reflector plays different roles on code and carrier multipath errors. Distant reflectors play dominant role for code and close-by reflectors play dominant role for the carrier. As a result the multipath error correlation time is different for the code and the carrier. In a complex multipath environment the correlation time is influenced by the dominant reflector in the environment.

It was also found that the multipath errors in L1 and L2 carriers have similar patterns, but different periods. Further investigations revealed that the code, carrier and the SNR multipath errors are phase related. The code and the SNR errors have similar phases and the carrier multipath is phase offset with respect to the code. This was further confirmed by the field data.

This analysis may be further extended using image theory of electromagnetic signals. The change in signal phase and polarization due to reflection, and its effect on various antennas, requires further research.

APPENDIX A: COMPUTATION OF MULTIPATH DELAY FROM A GEOMETRICAL PERSPECTIVE

The differential path delay of the multipath signal with respect to the direct signal can be obtained from a geometrical perspective as shown in Figure A1. The figure is similar to Figure 2, except that only one antenna and a single reflector case is considered here. This is one of the two scenarios that represent all the possible scenarios of the antenna-reflector geometry.

In Figure A1, the direct and reflected signals arrive at the antenna at point O. P1 is the point of reflection and P11 is its footprint on the XY plane. The solid lines in the figure are GPS signals. The dotted lines are on the XY plane and the dashed lines are either slant or vertical. The dotted and dashed lines are drawn for the purpose of analysis only. They are described as follows:

1. Draw a dotted line (OP12) perpendicular to the projection of the direct signal to the antenna on the XY plane (CO).
2. Draw a dotted line (P11P12), which is a projection of the direct signal to the reflector on the XY plane (AP1). This intersects the line drawn in step 1 at point P12.
3. Draw a dashed line (P11P13) from P11 with an elevation angle of the direct signal and in the same vertical plane on which the direct signal to the reflector lie.

4. Draw a dashed line (P12P15) from point P12 and perpendicular to the XY plane. This intersects the line drawn in step 3 at P13. This also intersects the direct signal to the reflector at point P15. Then P15P1 and P13P11 are parallel and of equal length.
5. Draw a dashed line (P12P14) from point P12, which intersects the direct signal to the reflector at point P14 orthogonally.

From the figure it can be observed that the plane containing the points O, P12 and P14 is a wavefront of the direct signal. Therefore, at any point on this plane, the signal will have the same phase. Then the reflected signal relative path delay (or the multipath delay) is equal to $|P14P1+P1O|$.

Now, from the paralleolgram P11P13P15P1,

$$\begin{aligned}
 |P15P1| &= |P13P11| \\
 &= \frac{|P12P11|}{\cos \theta}
 \end{aligned} \tag{A1}$$

From the triangle P11OP12,

$$\angle OP12P11 = 90^\circ \tag{A2}$$

$$\angle P11OP12 = \varphi_1 - (\varphi + 90) \tag{A3}$$

$$|OP11| = d_1 \tag{A4}$$

Then,

$$\begin{aligned}
 |P12P11| &= |OP11| \sin(\angle P11OP12) \\
 &= d_1 \sin(\varphi_1 - (\varphi + 90))
 \end{aligned}$$

$$= d_1 \cos(\varphi - \varphi_1) \quad (\text{A5})$$

$$\text{Therefore, from E.1 and E.5, } |P15P1| = \frac{d_1 \cos(\varphi - \varphi_1)}{\cos \theta} \quad (\text{A6})$$

Now, in the triangle P12P15P14

$$\angle P15P14P12 = 90^\circ \quad (\text{A7})$$

$$\angle P14P12P15 = \theta \quad (\text{A8})$$

$$\begin{aligned} |P12P15| &= |P12P13| + |P13P15| \\ &= |P12P13| + |P11P1| \\ &= |P11P12| \tan \theta + d_1 \tan \theta_1 \\ &= d_1 \cos(\varphi - \varphi_1) \tan \theta + d_1 \tan \theta_1 \end{aligned} \quad (\text{A9})$$

Therefore,

$$\begin{aligned} |P15P14| &= |P12P15| \sin \theta \\ &= \{d_1 \cos(\varphi - \varphi_1) \tan \theta + d_1 \tan \theta_1\} \sin \theta \end{aligned} \quad (\text{A10})$$

Therefore, the differential path delay

$$\begin{aligned} |P14P1| + |P1O| &= |P15P1| - |P15P14| + |P1O| \\ &= \frac{d_1 \cos(\varphi - \varphi_1)}{\cos \theta} - (d_1 \cos(\varphi - \varphi_1) \tan \theta + d_1 \tan \theta_1) \sin \theta + \frac{d_1}{\cos \theta_1} \\ &= \frac{d_1 \cos(\varphi - \varphi_1)}{\cos \theta} - \frac{d_1 \cos(\varphi - \varphi_1) \sin^2 \theta}{\cos \theta} - d_1 \tan \theta_1 \sin \theta + \frac{d_1}{\cos \theta_1} \\ &= d_1 \left\{ \frac{1}{\cos \theta_1} - \tan \theta_1 \sin \theta - \cos \theta \cos(\varphi - \varphi_1) \right\} \end{aligned} \quad (\text{A11})$$

The same result is obtained for the second reflector shown in Figure 2.

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